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Characters of cuspidal unramified series for central simple algebras of prime degree

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INTRODUCTION

Let A be a central simple algebra of dimension n^2 over a non-archimedean local field F and L be a a maximal unramified extension of F in A. Gerardin [G] constructed an irreducible supercuspidal representation π_θ of $A^\times$ associated with a regular quasi-character θ of $L^\times$. (θ is regular $\Longleftrightarrow \theta^\sigma \neq \theta\ \forall\sigma \in \mathrm{Gal}(L/F)$).

The aim of this article is to get the character formula of π_θ on regular elements in all compact modulo center Cartan subgroups of $A^\times$ when $[A:F] = l^2$, l an odd prime. (For the case $l = 2$, see [**HSY**]). We note that, when l is a prime, A is isomorphic to the division algebra of dimension l^2 over F or the algebra of $l \times l$ matrices over F. Our character formula is as follows.

THEOREM. *Let θ be a regular quasi-character of $L^\times$ with $\min_\eta f(\theta \otimes (\eta \circ N_{L/F})) = m+1$ and $\Gamma = \mathrm{Gal}(L/F)$. ($f(\theta) = \min\{n |\, \mathrm{Ker}\,\theta \supset 1 + P_L^n\}$). We denote by χ_{π_θ} the character of π_θ. Let x be an elliptic regular element in $A^\times$.*

(1) *If $F(x) = L$, then*

$$\chi_{\pi_\theta}(x) = \begin{cases} q^{\frac{l(l-1)j}{2}} \left(\sum_{\sigma\in\Gamma} \theta(x^\sigma) \right) & \text{if} \quad x \in U_j^* \quad (0 \le j < m) \\ q^{\frac{l(l-1)m}{2}} \left(\sum_{\sigma\in\Gamma} \theta(x^\sigma) \right) & \text{if} \quad x \in U_m. \end{cases}$$

where $U_0 = L^\times, U_i = F^\times(1+P_L^i)$ $(i \ge 1)$ and $U_i^* = U_i - U_{i+1}$.

(2) *If $F(x) \not\simeq L$, then*

$$\chi_{\pi_\theta}(x) = \begin{cases} 0 & \text{if} \quad x \notin F^\times(1 + P_{F(x)}^{lm+1}) \\ \theta(c) l q^{\frac{l(l-1)m}{2}} & \text{if} \quad x = c(1+y) \in F^\times(1+P_{F(x)}^{lm+1}). \end{cases}$$

Remark. (a) Any compact (mod center) Cartan subgroup of $A^\times$ is isomorphic to $E^\times$ for some extension E/F of degree n. Therefore the above formula gives the complete information on the set of elliptic regular elements of $A^\times$.
(b) For the case $F(x) = L$, the above formula can be written as follows:

$$\chi_{\pi_\theta}(x) = \Delta(x)^{-1} \sum_{\sigma\in W(L^\times)} \theta(x^\sigma) \quad \text{if} \quad x \in U_j^* \quad (0 \le j < m).$$

where $\Delta(x) = |det(Ad(x) - 1)_{A/L}|_F^{\frac{1}{2}}$ and $W(L^\times)$ is the Weyl group with respect to the Cartan subgroup $L^\times$. This is the analogy of the following formulas:

(1) character formula for irreducible square-integrable representations of real semisimple Lie groups (see [**HC**]);
(2) character formula for principal series induced from a regular character of a maximal split torus;
(3) character formula for irreducible unitary representations of compact Lie groups (Weyl's character formula).

In this article, we shall prove the formula when A is a division algebra. For the matrix algebra case, we use the result of division algebra case and Deligne-Kazhdanabstract matching theorem ([**BDKV**]): there is a bijection between irreducible representations of $D_n^\times$ and essentially square-integrable representations of $\mathrm{GL}_n(F)$ which preserves the characters up to $(-1)^{n-1}$ (D^n is a division algebra of dimension n^2 over F). Then we have only to calculate the character only on the set of 'very cuspidal' elements. More precisely, see [**T**].

We denote by $\mathcal{O}_F$, P_F, ϖ_F,k_F and v_F the maximal order of F, the maximal ideal of $\mathcal{O}_F$, a prime element of P_F, the residue field of F and the valuation of F normalized by $v_F(\varpi_F) = 1$. We set q be the number of elements in k_F. Hereafter we fix an additive character ψ of F whose conductor is P_F i.e. ψ is trivial on P_F and not trivial on $\mathcal{O}_F$. For an irreducible admissible representation π of $A^\times$, the conductoral exponent of π is defined to be the integer $f(\pi)$ such that the local constant $\epsilon(s,\pi,\psi)$ of Godement-Jacquet [**GJ**] is the form $aq^{-s(f(\pi)-n)}$ where $n^2 = [A : F]$. We call π *minimal* if

$$f(\pi) = \min_{\eta} f(\pi \otimes (\eta \circ N_{A/F}))$$

where η runs through the quasi-characters of $F^\times$. For a quasi-character η of $F^\times$, $\eta \circ N_{A/F}$ is denoted by simply η when there is no risk of confusion. Let G be a totally disconnected, locally compact group. We denote by $\widehat{G}$ the set of (equivalence classes of) irreducible admissible representations of G.

1. Construction of the representation. Let D be a division algebra of degree l (dimension l^2) over F with $l$an odd prime. We denote by $\mathcal{O}_D$, P_D, ϖ_D and v_D the maximal order of D, the maximal ideal of $\mathcal{O}_D$, a prime element of P_D and the valuation of D normalized by $v_D(\varpi_D) = 1$.

Let L be an unramified extension of F of degree l. L can be embedded into D and ,up to conjugacy, the embedding is unique.

DEFINITION 1.1. *Let θ be a quasi-character of $L^\times$.*

(1) *θ is called regular if all its conjugates by the action of $\mathrm{Gal}(L/F)$ are distinct. We denote by $\widehat{L}^\times_{reg}$ the set of regular quasi-characters of $L^\times$.*
(2) *Let $f(\theta) = \min\{n | \operatorname{Ker}\theta \supset 1 + P_L^n\}$. θ is called generic if either*
    (*a*) *$f(\theta) = 1$ and θ is not written in the form $\eta \circ N_{L/F}$ where η is a quasi-character of $F^\times$ or*
    (*b*) *$f(\theta) > 1$ and $k_F(\varpi^{f(\theta)-1}\gamma_\theta) = k_L$ where $\gamma_\theta \in P_L^{1-f(\theta)} - P_L^{2-f(\theta)}$ such that $\theta(1+x) = \psi(\mathrm{tr}_{L/F}(\gamma_\theta x))$ for $x \in P_L^{f(\theta)-1}$.*

We note that any regular quasi-character of $L^\times$ is written in the form $(\eta \circ N_{L/F}) \otimes \theta$ where η is a quasi-character of $F^\times$ and θ is a generic quasi-character of $L^\times$.

We construct an irreducible representation π_θ from $\theta \in \widehat{L}^\times_{reg}$ according to [**G**]. At first we treat the case θ is generic. If $f(\theta) = 1$, then θ itself can be regarded as a quasi-character of $F^\times \mathcal{O}_D^\times$ since $F^\times \mathcal{O}_D^\times / 1 + P_D \simeq L^\times / 1 + P_L$. Therefore we set

$$\pi_\theta = \mathrm{Ind}_{F^\times \mathcal{O}_D^\times}^{D^\times} \theta. \tag{1.2}$$

Then π_θ is an irreducible representation of $D^\times$ with $f(\pi_\theta) = l$. If $f(\theta) = m + 1 > 1$, then there exists an element $\gamma_\theta \in P_L^{-m} - (F \cap P_L^{-m}) + P_L^{1-m}$ such that

$$\theta(1 + x) = \psi(\mathrm{tr}_{L/F}(\gamma_\theta x)) \quad \text{for} \quad x \in P_L^{[\frac{m+2}{2}]} \tag{1.3}$$

where [ ] is the greatest integer function. (Recall that the conductor of ψ is P_F.) Let $\psi_{\gamma_\theta}(1 + x) = \psi(\mathrm{tr}_{D/F}(\gamma_\theta x))$ for $x \in P_D^{[\frac{ml+2}{2}]}$. Then ψ_{γ_θ} is a quasi-character of $1 + P_D^{[\frac{ml+2}{2}]}$. Set $H = L^\times(1 + P_D^{[\frac{ml+2}{2}]}) \subset D^\times$ and define a quasi-character ρ_θ of H by

$$\rho_\theta(h \cdot g) = \theta(h)\psi_{\gamma_\theta}(g) \qquad \text{for} \quad h \in L^\times, \quad g \in 1 + P_D^{[\frac{ml+2}{2}]}. \tag{1.4}$$

We set

$$\pi_\theta = \mathrm{Ind}_H^{D^\times} \rho_\theta. \tag{1.5}$$

Then π_θ is an irreducible minimal representation of $D^\times$ with $f(\pi_\theta) = l(m+1)$. (cf. [**H**],IV).

For a regular quasi-character θ written in the form $\theta = (\eta \circ N_{L/F}) \otimes \theta'$ where η is a quasi-character of $F^\times$ and θ' is a non-trivial generic quasi-character of $L^\times$, we set

$$\pi_\theta = \pi_{\theta'} \otimes \eta. \tag{1.6}$$

Now we get a correspondence $\theta \in \widehat{L}^\times_{reg} \mapsto \pi_\theta \in \widehat{D}^\times$. The following result is known about this correspondence. (cf. [**G**],[**H**]).

PROPOSITION 1.7. *With the above notations, for any regular quasi-character θ of $L^\times$, π_θ is an irreducible representation of $D^\times$ such that:*

(*a*) *the representations π_θ and $\pi_{\theta'}$ associated two regular quasi-characters θ and θ' are equivalent if and only if θ and θ' are conjugate under* Gal(L/F)*;*
(*b*) *the central quasi-character of π_θ is the restriction of θ to $F^\times$;*
(*c*) *for any quasi-character η of $F^\times$, the twisted representation of $\pi_\theta \otimes \eta$ is equivalent to $\pi_{\theta \otimes \eta \circ N_{L/F}}$;*
(*d*) *the contagredient representation of π_θ is equivalent to $\pi_{\theta^{-1}}$;*
(*e*) *the L-function of π_θ is* 1*;*
(*f*) *the ϵ-factor of π_θ is $\epsilon(\pi_\theta, \psi) = \epsilon(\theta, \psi \circ \mathrm{tr}_{L/F})$; in particular $f(\pi_\theta) = l \cdot f(\theta)$;*
(*g*) $\{\pi_\theta | \theta \in \widehat{L}^\times_{reg}\} = \{\pi \in \widehat{D}^\times | f(\pi) \equiv 0 \pmod{l}\}$.

2. Character formula. In this subsection we compute the character of π_θ. More precisely, for a separable extension E/F of degree l in D/F, we give the decomposition of π_θ as $E^\times$ module . First we treat the case E is unramified. We can assume $E = L$ because E is conjugate to L in D. Let $U_0 = L^\times, U_i = F^\times(1 + P_L^i)$ $(i \geq 1)$, $U_i^* = U_i - U_{i+1}$ and $X_i = \bigoplus_{\chi \in (L^\times/U_i)^\wedge} \chi$. We set $\Gamma = \mathrm{Gal}(L/F)$ and denote by χ_{π_θ} the character of π_θ.

THEOREM 2.1. *Let θ be a generic quasi-character of $L^\times$ with $f(\theta) = m + 1$ and π_θ as in* (1.2) *and* (1.5).

(1) (*Decomposition of π_θ as $L^\times$-module*)

$$\pi_\theta|_{L^\times} = (\bigoplus_{\sigma \in \Gamma} \theta \circ \sigma) \bigotimes \left(X_0 + (q-1) \frac{q^{\frac{l(l-1)}{2}} - 1}{q^l - 1} \sum_{a=1}^{m} q^{\frac{(l-1)(l-2)(a-1)}{2}} X_a \right).$$

(2) (*Character formula of π_θ on $L^\times$*)

$$\chi_{\pi_\theta}(x) = \begin{cases} q^{\frac{l(l-1)j}{2}} \left(\sum_{\sigma \in \Gamma} \theta(x^\sigma) \right) & \textit{if} \quad x \in U_j^* \quad (0 \leq j < m) \\ q^{\frac{l(l-1)m}{2}} \left(\sum_{\sigma \in \Gamma} \theta(x^\sigma) \right) & \textit{if} \quad x \in U_m. \end{cases}$$

COROLLARY 2.2. *Let θ be a regular quasi-character of $L^\times$ with $\min_\eta f(\theta \otimes (\eta \circ N_{L/F})) = m + 1$ and π_θ as in* (1.6).

(1) (*Decomposition of π_θ as $L^\times$-module*)

$$\pi_\theta|_{L^\times} = (\bigoplus_{\sigma \in \Gamma} \theta \circ \sigma) \bigotimes \left(X_0 + (q-1) \frac{q^{\frac{l(l-1)}{2}} - 1}{q^l - 1} \sum_{a=1}^{m} q^{\frac{(l-1)(l-2)(a-1)}{2}} X_a \right).$$

(2) (*Character formula of π_θ on $L^\times$*)

$$\chi_{\pi_\theta}(x) = \begin{cases} q^{\frac{l(l-1)j}{2}} \left(\sum_{\sigma \in \Gamma} \theta(x^\sigma) \right) & \textit{if} \quad x \in U_j^* \quad (0 \leq j < m) \\ q^{\frac{l(l-1)m}{2}} \left(\sum_{\sigma \in \Gamma} \theta(x^\sigma) \right) & \textit{if} \quad x \in U_m. \end{cases}$$

PROOF OF COROLLARY 2.2: This follows immediately from Proposition 1.7 (c) and Theorem 2.1.

We need several steps to prove Theorem 2.1. Let us start with the structure of D. By Skolem-Noether theorem, there exists a prime element $\xi \in \mathcal{O}_D$ such that

$$\xi^{-1} x \xi = x^\sigma \quad \text{for any} \quad x \in L,$$

where σ is a generator of $\mathrm{Gal}(L/F)$. We set $\varpi = \xi^l$. Then it follows that ϖ is a prime element of $\mathcal{O}_F$ and

$$
\begin{array}{lllllllll}
D & = L & \oplus & \xi L & \oplus & \cdots & \oplus \xi^{l-1} L \\
\mathcal{O}_D & = \mathcal{O}_L & \oplus & \xi \mathcal{O}_L & \oplus & \cdots & \oplus \xi^{l-1} \mathcal{O}_L \\
P_D & = P_L & \oplus & \xi \mathcal{O}_L & \oplus & \cdots & \oplus \xi^{l-1} \mathcal{O}_L \\
 & \vdots & & & & & \\
P_D^{l-1} & = P_L & \oplus & \xi P_L & \oplus & \cdots & \oplus \xi^{l-1} \mathcal{O}_L .
\end{array} \tag{2.3}
$$

Let θ be a generic quasi-character of $L^\times$ with $f(\theta) = m+1$. If $f(\theta) = 1$, then $\pi_\theta = \mathrm{Ind}_{F^\times \mathcal{O}_D^\times}^{D^\times} \theta$. Since $\{1, \xi, \xi^2, \cdots, \xi^{l-1}\}$ is a complete system of representatives of $D^\times / F^\times \mathcal{O}_D^\times$, we get $\chi_{\pi_\theta} = \sum_{\sigma \in \Gamma} (\theta \circ \sigma)$. We assume $f(\theta) = m+1 > 1$. We recall that $\pi_\theta = \mathrm{Ind}_H^{D^\times} \rho_\theta$, where $H = L^\times (1 + P_D^{[\frac{ml+2}{2}]})$. (See (1.4) for the definition of ρ_θ). It follows from (2.3) that

$$
H = F^\times (\mathcal{O}_L^\times + \xi P_L^{[\frac{m+1}{2}]} + \cdots + \xi^{\frac{l-1}{2}} P_L^{[\frac{m+1}{2}]} + \xi^{\frac{l+1}{2}} P_L^{[\frac{m}{2}]} + \cdots + \xi^{l-1} P_L^{[\frac{m}{2}]}). \tag{2.4}
$$

By Mackey decomposition [**S**],

$$
\pi_\theta |_{L^\times} = \bigoplus_{a \in L^\times \backslash D^\times / H} \mathrm{Ind}_{aHa^{-1} \cap L^\times}^{L^\times} \rho_\theta^a, \tag{2.5}
$$

where $\rho_\theta^a(x) = \rho_\theta(a^{-1} x a)$ for $x \in aHa^{-1} \cap L^\times$.

At first, we shall investigate $L^\times \backslash D^\times / H$. We have only to consider $L^\times \backslash F^\times \mathcal{O}_D^\times / H$ because

$$
L^\times \backslash D^\times / H = \bigcup_{i=0}^{l-1} \xi^i (L^\times \backslash F^\times \mathcal{O}_D^\times / H) \qquad \text{(disjoint union)}. \tag{2.6}
$$

For convenience, we often use the following notation:

$$
n(i) = \begin{cases} [\frac{m+1}{2}] & (1 \le i \le \frac{l-1}{2}) \\ [\frac{m}{2}] & (\frac{l+1}{2} \le i \le l-1). \end{cases} \tag{2.7}
$$

LEMMA 2.8. *Let $a = 1 + \sum_{i=1}^{l-1} \xi^i \alpha_i$ and $b = 1 + \sum_{i=1}^{l-1} \xi^i \beta_i$ $(\alpha_i, \beta_i \in \mathcal{O}_L)$. Then $aH = bH$ if and only if $\alpha_i - \beta_i \in P_L^{n(i)}$ for $1 \le i \le l-1$.*

PROOF: By (2.4), $aH = bH$ implies that there exist $\gamma_0 \in \mathcal{O}_L^\times$ and $\gamma_1, \cdots, \gamma_{l-1} \in P_L^{n(i)}$ such that $b = a(\sum_{i=0}^{l-1} \xi^i \gamma_i)$. Since $\mathcal{O}_D = \mathcal{O}_L \oplus \xi \mathcal{O}_L \oplus \cdots \oplus \xi^{l-1} \mathcal{O}_L$ and $\xi^{-1} x \xi = x^\sigma$ for $x \in L$, we obtain:

$$
1 = \gamma_0 + \varpi \sum_{j=1}^{l-1} \gamma_j \alpha_{l-j}^{\sigma^j}
$$

$$
\begin{aligned}
\beta_i - \alpha_i = (\gamma_0 - 1) + \gamma_i + \sum_{j=1}^{i-1} \gamma_j \alpha_{i-j}^{\sigma^j} \\
+ \varpi \sum_{j=i+1}^{l-1} \gamma_j \alpha_{l+i-j}^{\sigma^j} \quad (1 \le i \le l-1).
\end{aligned} \tag{$*$}
$$

Therefore we have $\gamma_0 \in 1 + P_L^{[\frac{m}{2}]+1}$ and $\beta_i - \alpha_i \in P_L^{n(i)}$ $(1 \le i \le l-1)$.

Conversely we assume $\beta_i - \alpha_i \in P_L^{n(i)}$ $(1 \le i \le l-1)$. By putting $\gamma_0 - 1 = -\varpi \sum_{j=1}^{l-1} \gamma_j \alpha_{l-j}^{\sigma^j}$ into $(*)$, we get

$$\beta_i - \alpha_i = (1 - \varpi\alpha_{l-i}^{\sigma^i})\gamma_i + \sum_{j=1}^{i-1} \gamma_j(\alpha_{i-j}^{\sigma^j} - \varpi\alpha_{l-j}^{\sigma^j}) + \varpi \sum_{j=i+1}^{l-1} \gamma_j(\alpha_{l+i-j}^{\sigma^j} - \alpha_{l-j}^{\sigma^j}) \quad (1 \le i \le l-1).$$

Thus it follows that

$$\begin{aligned} v_L(\gamma_i) &\ge \min([\tfrac{m+1}{2}], v_L(\gamma_1), \cdots, v_L(\gamma_{i-1}), v_L(\gamma_{i+1}) + 1, \cdots, v_L(\gamma_{l-1}) + 1) \\ &\qquad\qquad\qquad\qquad \text{for } \ 1 \le i \le \tfrac{l-1}{2}, \\ v_L(\gamma_i) &\ge \min([\tfrac{m}{2}], v_L(\gamma_1), \cdots, v_L(\gamma_{i-1}), v_L(\gamma_{i+1}) + 1, \cdots, v_L(\gamma_{l-1}) + 1) \\ &\qquad\qquad\qquad\qquad \text{for } \ \tfrac{l+1}{2} \le i \le l-1. \end{aligned}$$

Hence our lemma follows from the following simple fact that there is no solution to the system of inequations:

$$x_i \ge \min(x_1, \cdots, x_{i-1}, x_{i+1} + 1, \cdots, x_{l-1} + 1) \quad (1 \le i \le l-1).$$

LEMMA 2.9. *We put*

$$M = \{(\alpha^\sigma\alpha^{-1}, \alpha^{\sigma^2}\alpha^{-1}, \cdots, \alpha^{\sigma^{l-1}}\alpha^{-1}) | \alpha \in L^\times\} \subset \mathcal{O}_L^{(1)} \times \cdots \times \mathcal{O}_L^{(1)} = (\mathcal{O}_L^{(1)})^{l-1},$$

where $\mathcal{O}_L^{(1)} = \operatorname{Ker} N_{L/F}$. *Then the map* $(\alpha_i) \in (\mathcal{O}_L)^{l-1} \mapsto 1 + \sum_{i=1}^{l-1} \xi^i \alpha_i \in \mathcal{O}_D^\times$ *induces a bijection from* $M \backslash (\mathcal{O}_L)^{l-1} / (P_L^{[\frac{m+1}{2}]})^{\frac{l-1}{2}} \times (P_L^{[\frac{m}{2}]})^{\frac{l-1}{2}}$ *to* $L^\times \backslash F^\times \mathcal{O}_D^\times / H$.

PROOF: For $\alpha \in L^\times$ and $\beta_1, \cdots .\beta_{l-1} \in \mathcal{O}_L$,

$$\alpha(1 + \sum_{i=1}^{l-1} \xi^i \beta_i)H = (1 + \sum_{i=1}^{l-1} \xi^i \alpha^{\sigma^i} \alpha^{-1} \beta_i)H.$$

Therefore our lemma is obtained from Lemma 2.8.

In order to prove Theorem 2.1, we need more information about $L^\times \backslash F^\times \mathcal{O}_D^\times / H$. We prepare some notations.

For $1 \leq i \leq l-1$ and $0 \leq \mu < n(i)$, we set

$$I_{\mu,i} = \begin{cases} M\backslash(\mathcal{O}_L)^{i-1} \times \mathcal{O}_L^\times \times (\mathcal{O}_L)^{l-i-1}/(P_L^{[\frac{m+1}{2}]-\mu-1})^{i-1} \times (1+P_L^{[\frac{m+1}{2}]-\mu}) \times \\ \qquad (P_L^{[\frac{m+1}{2}]-\mu})^{\frac{l-1}{2}-i} \times (P_L^{[\frac{m}{2}]-\mu})^{\frac{l-1}{2}} \quad \text{for} \quad 1 \leq i \leq \frac{l-1}{2}, \\ M\backslash(\mathcal{O}_L)^{i-1} \times \mathcal{O}_L^\times \times (\mathcal{O}_L)^{l-i-1}/(P_L^{[\frac{m+1}{2}]-\mu-1})^{\frac{l-1}{2}} \times (P_L^{[\frac{m}{2}]-\mu-1})^{i-\frac{l+1}{2}} \times \\ \qquad (1+P_L^{[\frac{m}{2}]-\mu}) \times (P_L^{[\frac{m}{2}]-\mu})^{l-1-i} \quad \text{for} \quad \frac{l+1}{2} \leq i \leq l-1, \end{cases}$$

$$J_{\mu,i} = \begin{cases} (\mathcal{O}_L/P_L^{[\frac{m+1}{2}]-\mu-1})^{i-1} \times (\mathcal{O}_F^\times/1+P_F^{[\frac{m+1}{2}]-\mu}) \times (\mathcal{O}_L/P_L^{[\frac{m+1}{2}]-\mu})^{\frac{l-1}{2}-i} \times \\ \qquad (\mathcal{O}_L/P_L^{[\frac{m}{2}]-\mu})^{\frac{l-1}{2}} \quad \text{for} \quad 1 \leq i \leq \frac{l-1}{2}, \\ (\mathcal{O}_L/P_L^{[\frac{m+1}{2}]-\mu-1})^{\frac{l-1}{2}} \times (\mathcal{O}_L/P_L^{[\frac{m}{2}]-\mu-1})^{i-\frac{l+1}{2}} \times (\mathcal{O}_F^\times/1+P_F^{[\frac{m}{2}]-\mu}) \times \\ \qquad (\mathcal{O}_L/P_L^{[\frac{m}{2}]-\mu})^{l-1-i} \quad \text{for} \quad \frac{l+1}{2} \leq i \leq l-1, \end{cases}$$

and

$$K_{\mu,i} = \{\, 1+\varpi^\mu(\sum_{j=1}^{i-1} \varpi\xi^j\beta_j + \sum_{j=i}^{l-1} \xi^j\beta_j) \mid \ (\beta_1, \cdots, \beta_{l-1}) \in I_{\mu,i}\}.$$

We define $\varphi_i \colon (\mathcal{O}_L)^{i-1} \times \mathcal{O}_L^\times \times (\mathcal{O}_L)^{l-i-1} \to (\mathcal{O}_L)^{i-1} \times \mathcal{O}_F^\times \times (\mathcal{O}_L)^{l-i-1}$ as follows:

$$\varphi_i(\alpha_1, \cdots, \alpha_{l-1}) = (\beta_1, \cdots, \beta_{l-1}), \quad \beta_j = \alpha_j\alpha_i^{\sigma^{-i}}\alpha_i^{\sigma^{-2i}} \cdots \alpha_i^{\sigma^{-ki}}, \tag{2.10}$$

where k is determined by $0 \leq k < l$ and $-ki \equiv j \pmod{l}$. (In particular $\beta_i = N_{L/F}\alpha_i$).

LEMMA 2.11. (1) *A complete system of representatives of the double coset* $L^\times\backslash F^\times\mathcal{O}_D^\times/H$ *is given by* $\bigcup_{\substack{1 \leq i \leq l-1 \\ 0 \leq \mu < n(i)}} K_{\mu,i} \cup \{1\}$.

(2) *The map* φ_i *induces a bijection from* $I_{\mu,i}$ *to* $J_{\mu,i}$.

PROOF: Part one follows immediately from Lemma 2.9. For part two, it suffices to see that φ_1 induces a bijection from $I_{0,1}$ to $J_{0,1}$. If $\beta_1, \gamma_1 \in \mathcal{O}_L^\times$ and $\beta_2, \cdots, \beta_{l-1}, \gamma_2, \cdots, \gamma_{l-1} \in \mathcal{O}_L$ satisfy $(\gamma_1, \cdots, \gamma_{l-1}) \in M(\beta_1, \cdots, \beta_{l-1})((1+P_L^{[\frac{m+1}{2}]}) \times (P_L^{[\frac{m+1}{2}]})^{\frac{l-3}{2}} \times (P_L^{[\frac{m}{2}]})^{\frac{l-1}{2}})$, then there exist $\alpha \in \mathcal{O}_L^\times$ and $y_i \in P_L^{n(i)}$ $(1 \leq i \leq l-1)$ such that

$$\gamma_1 = \alpha^\sigma\alpha^{-1}\beta_1(1+y_1),$$
$$\gamma_i = \alpha^{\sigma^i}\alpha^{-1}(\beta_i+y_i) \quad (2 \leq i \leq l-1).$$

This implies:

$$N_{L/F}(\beta_1) \equiv N_{L/F}(\gamma_1) \mod 1+P_L^{[\frac{m+1}{2}]} \quad \text{(multiplicative equivalence)},$$
$$\gamma_i\gamma_1^{\sigma^{-1}} \cdots \gamma_1^{\sigma^i} \equiv \beta_i\beta_1^{\sigma^{-1}} \cdots \beta_1^{\sigma^i} \mod P_L^{n(i)} \quad \text{for} \quad 2 \leq i \leq l-1.$$

Therefore φ_1 induces a well-defined map from $I_{0,1}$ to $J_{0,1}$. The induced map's bijectivity follows from the bijectivity of the map $\mathcal{O}_L^{(1)}\backslash\mathcal{O}_L^\times/1+P_L^j \xrightarrow{N_{L/F}} \mathcal{O}_F^\times/1+P_F^j$.

Next we consider the term $aHa^{-1} \cap L^\times$ in (2.5).

LEMMA 2.12. *If $a \in K_{\mu,i}$, then $aHa^{-1} \cap L^\times = F^\times(1+P_L^{n(i)-\mu})$.*

PROOF: Since $F^\times \subset aHa^{-1} \cap L^\times$, we have only to see $aHa^{-1} \cap \mathcal{O}_L^\times = \mathcal{O}_F^\times(1+P_L^{n(i)-\mu})$. If $\alpha \in aHa^{-1} \cap \mathcal{O}_L^\times$, then there exist $\gamma_0 \in \mathcal{O}_L^\times$ and $\gamma_i \in P_L^{n(i)-\mu}(1 \le i \le l-1)$ such that $\alpha a = a\sum_{i=0}^{l-1}\xi^i\gamma_i$. Put $a = 1+\sum_{j=1}^{l-1}\xi^j\beta_j$. Then we have

$$\gamma_0 = \alpha - \varpi\sum_{j=1}^{l-1}\gamma_i\beta_{l-j}^{\sigma^j},$$

$$(\alpha^{\sigma^{-i}} - \gamma_0)\beta_i = \gamma_i + \sum_{j=1}^{i}\beta_{i-j}^{\sigma^j}\gamma_j + \varpi\sum_{j=i+1}^{l-1}\beta_{l+i-j}^{\sigma^j}\gamma_j. \quad (1 \le i \le l-1).$$

By replacing γ_0 by $\alpha - \varpi\sum_{j=1}^{l-1}\gamma_i\beta_{l-j}^{\sigma^j}$, we get

$$(\alpha^{\sigma^{-i}} - \alpha)\beta_i \in P_L^{n(i)} \qquad (1 \le i \le l-1).$$

Therefore $\alpha \in \mathcal{O}_F^\times(1+P_L^{n(i)-\mu})$ and $aHa^{-1}\cap\mathcal{O}_L^\times \subset \mathcal{O}_F^\times(1+P_L^{n(i)-\mu})$. As for $aHa^{-1}\cap\mathcal{O}_L^\times \supset \mathcal{O}_F^\times(1+P_L^{n(i)-\mu})$, we can prove it by the same argument in the proof of Lemma 2.8.

Our next task is to compute ρ_θ^a for $a \in L^\times\backslash D^\times/H$. The above lemma tells us that $\rho_\theta^a \in (F^\times(1+P_L^{n(i)-\mu}))^\wedge$ if $a \in K_{\mu,i}$. If $a' = \xi^j a$, then $a'Ha'^{-1} \cap L^\times = aHa^{-1}\cap L^\times$ and $\rho_\theta^{a'} = \rho_\theta^a \circ \sigma^j$. Therefore it suffices to consider ρ_θ^a for $a \in L^\times\backslash F^\times\mathcal{O}_D^\times/H$.

LEMMA 2.13. *Let $c \in F^\times, y \in P_L^{n(i)-\mu}$ and $a = 1+\varpi^\mu(\varpi\sum_{j=1}^{i-1}\xi^j\alpha_j + \sum_{j=i}^{l-1}\xi^j\alpha_j) \in K_{\mu,i}$. Then*

$$\begin{aligned}(\rho_\theta^a\rho_\theta^{-1})(c(1+y)) = \psi(\mathrm{tr}_{L/F}\,\varpi^{\mu+1}(\varpi\sum_{j=1}^{i-1}(\gamma_\theta{}^{\sigma^{-j}}f_{l-j}(a)\alpha_j^{\sigma^{-j}} - \gamma_\theta(f_{l-j}(a))^{\sigma^j}\alpha_j)\\ +\sum_{j=i}^{l-1}(\gamma_\theta{}^{\sigma^{-j}}f_{l-j}(a)\alpha_j^{\sigma^{-j}} - \gamma_\theta(f_{l-j}(a))^{\sigma^j}\alpha_j))y),\end{aligned}$$

where $f_j(a) \in L$ is defined by $a^{-1} = \sum_{i=0}^{l-1}\xi^j f_j(a)$.

PROOF: Since $(\rho_\theta^a\rho_\theta^{-1})$ is trivial on $F^\times$, we can assume $c = 1$. Put $g = 1+x$, then

$$\begin{aligned}a^{-1}gag^{-1} &= (1+a-1)^{-1}g(1+a-1)g^{-1}\\ &= (1+a-1)^{-1}(1+g(a-1)g^{-1})\\ &= 1+a^{-1}(g(a-1)g^{-1}-(a-1))\\ &= 1+a^{-1}\varpi^\mu(\varpi\sum_{j=1}^{i-1}\xi^j\alpha_j(g^{\sigma^j}g^{-1}-1)+\sum_{j=i}^{l-1}\xi^j\alpha_j(g^{\sigma^j}g^{-1}-1)).\end{aligned}$$

Since $\varpi^{\mu}(\varpi\sum_{j=1}^{i-1}\xi^j\alpha_j+\sum_{j=i}^{l-1}\xi^j\alpha_j)\in P_D^{[\frac{ml+2}{2}]}$, $\rho_\theta(1+x)=\psi(\mathrm{tr}_{D/F}\gamma_\theta x)$ $(x\in P_D^{[\frac{ml+2}{2}]})$ and $\mathrm{tr}_{D/F}\gamma_\theta\xi^jL=0 \quad (1\le j\le l-1)$,

$$\begin{aligned}(\rho_\theta^a\rho_\theta^{-1})(g)&=\rho_\theta(a^{-1}gag^{-1})\\&=\psi(\mathrm{tr}_{D/F}\gamma_\theta a^{-1}\varpi^{\mu}(\varpi\sum_{j=1}^{i-1}\xi^j\alpha_j(g^{\sigma^j}g^{-1}-1)\\&\qquad+\sum_{j=i}^{l-1}\xi^j\alpha_j(g^{\sigma^j}g^{-1}-1)))\\&=\psi(\mathrm{tr}_{L/F}\gamma_\theta a^{-1}\varpi^{\mu+1}(\varpi\sum_{j=1}^{i-1}(f_{l-j}(a))^{\sigma^j}\alpha_j(g^{\sigma^j}g^{-1}-1)\\&\qquad+\sum_{j=i}^{l-1}(f_{l-j}(a))^{\sigma^j}\alpha_j(g^{\sigma^j}g^{-1}-1))).\end{aligned}$$

In the last term of the above equations, $\gamma_\theta\in P_L^{-m}$, $f_{l-j}(a)\in P_L^{\mu}$ and $g^{\sigma^j}g^{-1}-1\equiv y^{\sigma^j}-y$ mod $P_L^{2(n(i)-\mu)}$. Therefore

$$\begin{aligned}(\rho_\theta^a\rho_\theta^{-1})(g)&=\psi(\mathrm{tr}_{L/F}\gamma_\theta a^{-1}\varpi^{\mu+1}(\varpi\sum_{j=1}^{i-1}(f_{l-j}(a))^{\sigma^j}\alpha_j(g^{\sigma^j}g^{-1}-1)\\&\qquad+\sum_{j=i}^{l-1}(f_{l-j}(a))^{\sigma^j}\alpha_j(g^{\sigma^j}g^{-1}-1))).\end{aligned}$$

(We note ψ is trivial on P_L). Hence our lemma follows from the following property:

$$\mathrm{tr}_{L/F}uv^{\sigma^j}=\mathrm{tr}_{L/F}u^{\sigma^{-j}}v \quad \text{for any} \quad u,v\in L.$$

We prepare the next lemma for the purpose of writing $f_k(a)$ by $(\alpha_j)_{1\le j\le l-1}$.

LEMMA 2.14. *For $a=\sum_{j=0}^{l-1}\xi^j\alpha_j$ $(\alpha_j\in L)$, put*

$$\begin{aligned}\Lambda(a)&=(\varpi^{[1+\frac{j-i}{l}]}\alpha_{i-j\bmod l})_{0\le i,j\le l-1}\\&=\begin{pmatrix}\alpha_0&\varpi\alpha_{l-1}^{\sigma}&\cdots&\varpi\alpha_1^{\sigma^{l-1}}\\\alpha_1&\alpha_0^{\sigma}&\ddots&\vdots\\\vdots&\ddots&\ddots&\varpi\alpha_{l-1}^{\sigma^{l-1}}\\\alpha_{l-1}&\cdots&\alpha_1^{\sigma^{l-2}}&\alpha_0^{\sigma^{l-1}}\end{pmatrix}\in M_l(L),\end{aligned}$$

and

$$\Lambda_k(a) = (-1)^k \begin{vmatrix} \alpha_1 & \cdots & \varpi\alpha_{l-k+1}^{\sigma^{k-1}} & \varpi\alpha_{l-k-1}^{\sigma^{k+1}} & \cdots & \varpi\alpha_2^{\sigma^{l-1}} \\ \vdots & & \vdots & \vdots & & \vdots \\ \alpha_k & \cdots & \alpha_1^{\sigma^{k-1}} & \varpi\alpha_{l-1}^{\sigma^{k+1}} & \cdots & \varpi\alpha_{k+1}^{\sigma^{l-1}} \\ \vdots & & \vdots & \vdots & & \vdots \\ \alpha_{l-1} & \cdots & \alpha_{l-k+2}^{\sigma^{k-1}} & \alpha_{l-k}^{\sigma^{k+1}} & \cdots & \alpha_0^{\sigma^{l-1}} \end{vmatrix} \in L^\times$$

i.e. $\Lambda_k(a)$ *is the* $(1, k+1)$*-cofactor of* $\Lambda(a)$*. Then*

$$a^{-1} = \sum_{j=0}^{l-1} \xi^j \frac{\Lambda_j(a)}{|\Lambda(a)|},$$

where $|\Lambda(a)|$ *is the determinant of* $\Lambda(a)$*.*

PROOF: By the map $\Lambda\colon D \to M_l(L)$, we can embed D into $M_l(L)$. Then our lemma follows from the basic matrix theory.

We define L-valued functions $R_{\mu,i}$ on $\mathcal{O}_L^{i-1} \times \mathcal{O}_L^\times \times \mathcal{O}_L^{l-i-1}$ by :

$$\begin{aligned} R_{\mu,i}(\beta_1, \cdots, \beta_{l-1}) = \varpi^{\mu+2} &\sum_{j=1}^{i-1} (\gamma_\theta^{\sigma^j} f_{l-j}(a)\alpha_j^{\sigma^j} - \gamma_\theta(f_{l-j}(a))^{\sigma^j}\alpha_j) \\ + \varpi^{\mu+1} &\sum_{j=i}^{l-1} (\gamma_\theta^{\sigma^j} f_{l-j}(a)\alpha_j^{\sigma^j} - \gamma_\theta(f_{l-j}(a))^{\sigma^j}\alpha_j), \end{aligned}$$

where $\varphi_i(\alpha_1, \cdots, \alpha_{l-1}) = (\beta_1, \cdots, \beta_{l-1})$ and $a = 1 + \varpi^\mu(\varpi \sum_{j=1}^{i-1} \xi^j \alpha_k + \sum_{j=i}^{l-1} \xi^j \alpha_k)$. (As for the definition of φ_i and $f_j(a)$, see 2.10 and Lemma 2.12 respectively). It is easily seen that $R_{\mu,i}$ is well-defined. In fact, we can show by virtue of Lemma 2.14 that $R_{\mu,i}(\beta_1, \cdots, \beta_{l-1})$ is a rational function of $\{\beta_j^{\sigma^k}\}_{1 \le j,k \le l-1}$. We fix $\beta_j (1 \le j \le l-1)$ for all j but $l-i$ and define a function $\tilde{R}_{\mu,i}$ on $\mathcal{O}_L$ by:

$$\tilde{R}_{\mu,i}(x) = R_{\mu,i}(\beta_1, \cdots, \beta_{l-i-1}, x, \beta_{l-i+1}, \cdots, \beta_{l-1}).$$

The next lemma is the key point in this proof of Theorem 2.1.

LEMMA 2.15. *Let* $L^{(0)} = \{x \in L \mid \mathrm{tr}_{L/F}\, x = 0\}$*. Then* $\tilde{R}_{\mu,i}$ *has the following property:*

(1) $\tilde{R}_{\mu,i}$ *induces a surjection from* $\mathcal{O}_L/P_L^{[\frac{m}{2}]-\mu}$ *to* $P_L^{2\mu+1-m} \cap L^{(0)}/P_L^{\mu+1-[\frac{m+1}{2}]} \cap L^{(0)}$ *and each fiber of the induced map has* $q^{[\frac{m}{2}]-\mu}$ *elements if* $1 \le i \le \frac{l-1}{2}$,

(2) $\tilde{R}_{\mu,i}$ *induces a surjection from* $\mathcal{O}_L/P_L^{[\frac{m+1}{2}]-\mu-1}$ *to* $P_L^{2\mu+2-m} \cap L^{(0)}/P_L^{\mu+1-[\frac{m}{2}]} \cap L^{(0)}$ *and each fiber of the induced map has* $q^{[\frac{m+1}{2}]-\mu-1}$ *elements if* $\frac{l+1}{2} \le i \le l-1$.

PROOF: We assume $1 \le i \le \frac{l-1}{2}$. By virtue of Lemma 2.14 and Lemma 2.15, we can show

$$\tilde{R}_{\mu,i}(x) \equiv ax - (ax)^{\sigma^i} + b \mod P_L^{v_L(x)+2\mu+1-m},$$

where $a = \varpi^{2\mu+1}(\gamma_\theta^{\sigma^{-i}} - \gamma_\theta) \in P_L^{2\mu+1-m} - P_L^{2\mu+2-m}$ and b is a constant in $P_L^{2\mu+1-m}$. Therefore we can get our lemma by induction on $[\frac{m}{2}] - \mu$ since $\tilde{R}_{\mu,i}(x) \mod P_L^{\mu+1-[\frac{m+1}{2}]}$ is a polynomial of $\{x, x^\sigma, \cdots, x^{\sigma^{l-1}}\}$ whose coefficients belong to $P_L^{2\mu+1-m}$. The case $\frac{l+1}{2} \le i \le l-1$ is proved by the same way.

Summing up the above lemmas, we have the following result.

LEMMA 2.16. (1) *If* $1 \le i \le \frac{l-1}{2}$,

$$\begin{array}{ccc} K_{\mu,i} & \to & (F^\times(1+P_L^{[\frac{m+1}{2}]-\mu}))^\wedge \\ a & \mapsto & \rho_\theta^a \rho_\theta^{-1} \end{array}$$

is a surjection to $\left(F^\times(1+P_L^{[\frac{m+1}{2}]-\mu})/F^\times(1+P_L^{m-2\mu})\right)^\wedge$ *and each fiber of the map has* $(q-1)q^{\frac{(l-1)(l-2)(m-2\mu)}{2}-l(i-1)-1}$ *elements.*

(2) *If* $\frac{l+1}{2} \le i \le l-1$,

$$\begin{array}{ccc} K_{\mu,i} & \to & (F^\times(1+P_L^{[\frac{m}{2}]-\mu}))^\wedge \\ a & \mapsto & \rho_\theta^a \rho_\theta^{-1} \end{array}$$

is a surjection to $\left(F^\times(1+P_L^{[\frac{m}{2}]-\mu})/F^\times(1+P_L^{m-2\mu-1})\right)^\wedge$ *and each fiber of the map has* $(q-1)q^{\frac{(l-1)(l-2)(m-2\mu-1)}{2}-l(i-\frac{l+1}{2})-1}$ *elements.*

PROOF: Let $1 \le s < t \le 2t, b \in P_L^s \cap L^{(0)}, c \in F^\times$ and $y \in P_L^{1-t}$. Then the map $b \mapsto \hat{b} = (c(1+y) \mapsto \psi(\mathrm{tr}_{L/F}\, by))$ induces an isomorphism between $P_L^s \cap L^{(0)}/P_L^t \cap L^{(0)}$ and $(F^\times(1+P_L^{1-t})/F^\times(1+P_L^{1-s}))^\wedge$ since the conductor of ψ is P_L and L/F is unramified. Hence our lemma holds by virtue of Lemma 2.15 and 2.12.

PROOF OF THEOREM 2.1: By Lemma 2.16,

$$\bigoplus_{a \in K_{\mu,i}} \mathrm{Ind}_{aHa^{-1}\cap L^\times}^{L^\times} \rho_\theta^a = \theta \bigotimes \begin{cases} (q-1)q^{\frac{(l-1)(l-2)(m-2\mu)}{2}-l(i-1)-1} X_{m-2\mu} \\ \qquad \text{if } 1 \le i \le \frac{l-1}{2}, \\ (q-1)q^{\frac{(l-1)(l-2)(m-2\mu-1)}{2}-l(i-\frac{l+1}{2})-1} X_{m-2\mu-1} \\ \qquad \text{if } \frac{l+1}{2} \le i \le l-1, \end{cases}$$

where $X_j = \bigoplus_{\chi \in (L^\times/F^\times(1+P_L^j))^\wedge} \chi$. Thus by Lemma 2.11 and (2.5), we have:

$$\pi_\theta|_{L^\times} = (\bigoplus_{\sigma\in\Gamma} \theta \circ \sigma) \bigotimes \left(X_0 + (q-1)\frac{q^{\frac{l(l-1)}{2}} - 1}{q^l - 1} \sum_{a=1}^{m} q^{\frac{(l-1)(l-2)(a-1)}{2}} X_a \right).$$

The rest of Theorem 2.1 follows immediately from the above formula.

Next we consider the case $E \not\subset L$. Then E is a totally ramified extension of F of degree l. This case is very easy.

THEOREM 2.17. *Let θ be a regular quasi-character of $L^\times$ with* $\min_\eta f(\theta\otimes(\eta\circ N_{L/F})) = m+1$ *and π_θ as in* (1.6).

(1) (*Decomposition of π_θ as $E^\times$-module*)

$$\pi_\theta|_{E^\times} = \theta \otimes q^{\frac{(l-1)(l-2)m}{2}} \bigoplus_{\chi\in(E^\times/F^\times(1+P_E^{lm+1}))^\wedge} \chi$$

(2) (*Character formula of π_θ on $E^\times$*)

$$\chi_{\pi_\theta}(x) = \begin{cases} 0 & \text{if } \; x \notin F^\times(1+P_E^{lm+1}) \\ \theta(c)lq^{\frac{l(l-1)m}{2}} & \text{if } \; x = c(1+y) \in F^\times(1+P_E^{lm+1}). \end{cases}$$

PROOF: It suffices to say that $\chi_{\pi_\theta}(x) = 0$ if $[\frac{lm+2}{2}] \le v_E(x-1) < lm$. (We note that $F^\times(1+P_E^{lm}) = F^\times(1+P_E^{lm+1})$). Set $r = v_E(x-1)$. From the definition of π_θ,

$$\begin{aligned}\chi_{\pi_\theta}(x) &= \sum_{g\in D^\times/H} \rho_\theta(g^{-1}xg) \\ &= \frac{1}{q^{l(lm+1-r-[\frac{lm+1-r}{2}])}} \sum_{g\in D^\times/H} \sum_{k\in P_D^{[\frac{lm+1-r}{2}]}/P_D^{lm+1-r}} \rho_\theta((1+k)^{-1}g^{-1}xg(1+k)).\end{aligned}$$

Set $g^{-1}xg = 1+h$. By virtue of $(1+k)^{-1}(1+h)(1+k) \equiv 1+hk-kh \mod P_D^{lm+1}$, $\rho_\theta((1+k)^{-1}(1+h)(1+k)) = \psi(\mathrm{tr}_{D/F}(\gamma_\theta h - h\gamma_\theta)k)$. Since $h \in P_D^r$ and $h \notin P_L^r + P_D^{r+1}$, the map $k \mapsto \psi(\mathrm{tr}_{D/F}(\gamma_\theta h - h\gamma_\theta)k)$ is a non-trivial character of $P_D^{[\frac{lm+1-r}{2}]}/P_D^{lm+1-r}$. (cf. 6.7 [**Ca**]). Therefore $\chi_{\pi_\theta}(x) = 0$.

REFERENCES

[BDKV] J. Bernstein, P. Deligne, D. Kazhdan and M. Vigneras, "Représentations des Groupes Réductifs sur un Corps Local," Herman, Paris, 1984.

[Ca] H. Carayol,, *Représentations cuspidales du groupe linéaire,*, Ann. Sci. Ec. Norm. Sup. **17** (1984), 191–226.

[HC] Harish-Chandra, *Harmonic analysis on semisimple Lie groups*, Bull.Amer. Math. Soc. **76** (1970), 529–551.

[HSY] H. Hijikata, H. Saito, M. Yamauchi, *Representations of quaternion algebras over local fields and trace formulas of Hecke operators*, preprint.

[G] P. Gerardin, *Cuspidal unramified series for central simple algebras*, in "Automorphic Forms, Representations, and L-functions," Proc. of Symp. in Pure Math., 33, Part 1, 1979, pp. 157–169.

[GJ] R. Godement and H. Jacquet, "Zeta functions of simple algebras," Springer Lecture Notes in Math. 260, 1972.

[H] R. Howe, *Kirillov theory for compact p-adic groups*, Pac. J. Math. **73** (1977), 365–381.

[M] A. Moy, *Local constant and the tame Langlands correspondence*, Amer. J. Math. **108** (1986), 863–930.

[Se] J. Serre, "Linear Representations of Finite Groups," Springer Verlag, New-York, 1977.

[T] T. Takahashi, *Characters of cuspidal unramified series for central simple algebras of prime degree*, to appear in J. of Math. of Kyoto Univ..